Grade 7/8 Math Circles

Feb 27/28, Mar 1/2

BCC and Gauss Contest Prep - Solutions

BCC Prep Solutions

Toy Storage

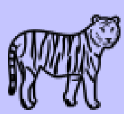
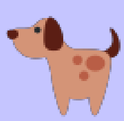
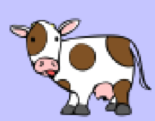
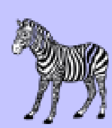
Tom has two types of toys: animal toys and vehicle toys. Tom fills three boxes by putting three toys in each box. As long as there is room, he puts

- vehicles into box A,
- animals with striped bodies into box B, and
- animals with spotted bodies into Box C.

However,

- Anytime he tries to put a toy into box A and it is full, he then tries to put the toy into box B.
- Anytime he tries to put a toy into box B and it is full, he then tries to put the toy into box C.
- Anytime he tries to put a toy into box C and it is full, he then tries to put the toy into box A.

Tom puts the following nine toys into boxes in the following order:

(1) 
(2) 
(3) 
(4) 
(5) 
(6) 
(7) 
(8) 
(9) 

Where does Tom put the dog and zebra?

A. Tom puts the dog in box C, and the zebra in box B.

B. Tom puts both in box A.

C. Tom puts both in box B.

D. Tom puts both in box C.

Solution

For toys (1) to (5), we can follow the rules and put them into their corresponding boxes.

When Tom tries to put the firetruck into box A, he realizes that it is full (because it has the taxi, the policecar and the car). He then puts it into box B which still has room for one more toy. At this point, box B has the tiger, the clownfish and the firetruck.

Tom can easily place the dog and the cow into box C as there is still room.

When Tom tries to put the zebra into box B, he realizes that it is full, so he places the zebra into box C.

So the answer is (D), Tom puts both the dog and the zebra into box C.

Swapping Dogs

Two types of dogs are standing as shown below.

A *swap* occurs when two dogs that are beside each other exchange positions. After some swaps, the three large dogs end up in three consecutive positions.

What is the fewest number of swaps that could have occured?

A. 5

B. 6

C. 7

D. 8

Solution

The answer is (B), the fewest number of swaps that could have occured is 6.

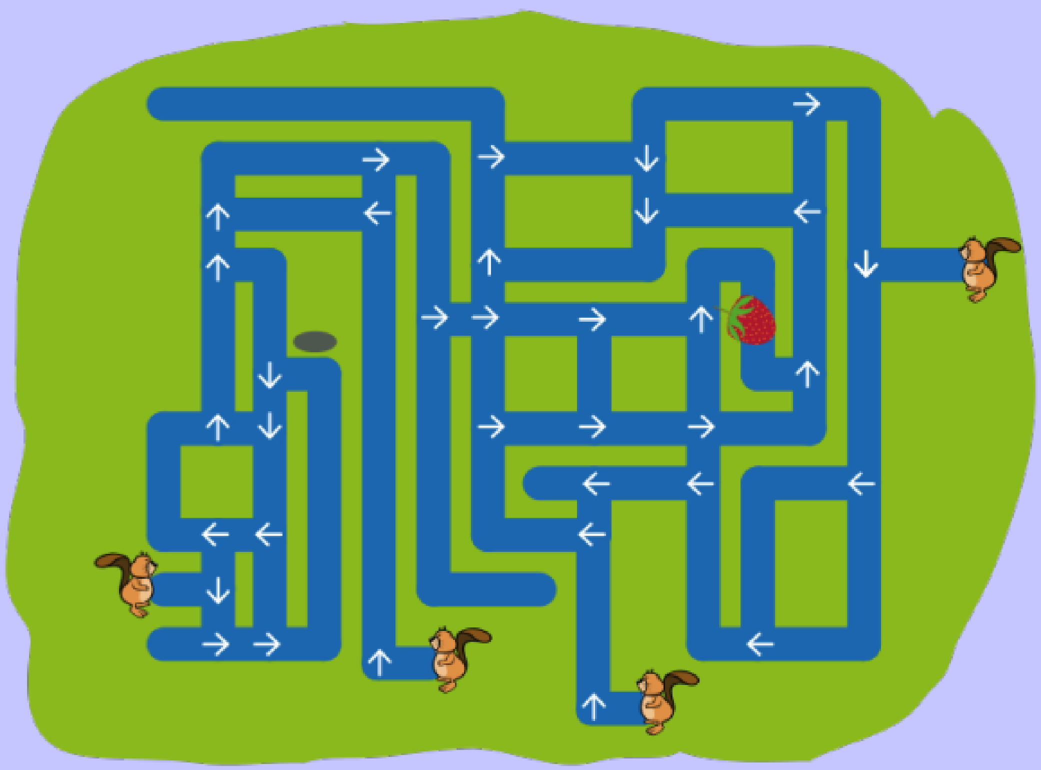
The three large dogs can end up in three consecutive positions by doing the following:

1. Swapping the large dog on the left twice towards the right, then
2. Swapping the large dog on the right four times towards the left.

When swapping, every small dog must be involved in a swap since there is at least one small dog between two large dogs initially. Further, each swap must include a small dog **and** a large dog since swapping two small dogs is not useful. There are six small dogs in total between any two large dogs, so there must be at least six swaps. As shown above, achieving six swaps is possible, and trying to move all three dogs to the left or to the right instead of the middle requires more than six swaps.

Strawberry Hunt

Four beavers swim through canals in an attempt to find a strawberry. They start at different places and always move in the direction of the arrows shown below.



Each beaver either finds the strawberry, swims in a loop forever, or reaches and remains at a dead-end.

How many beavers find the strawberry?

A. 1

B. 2

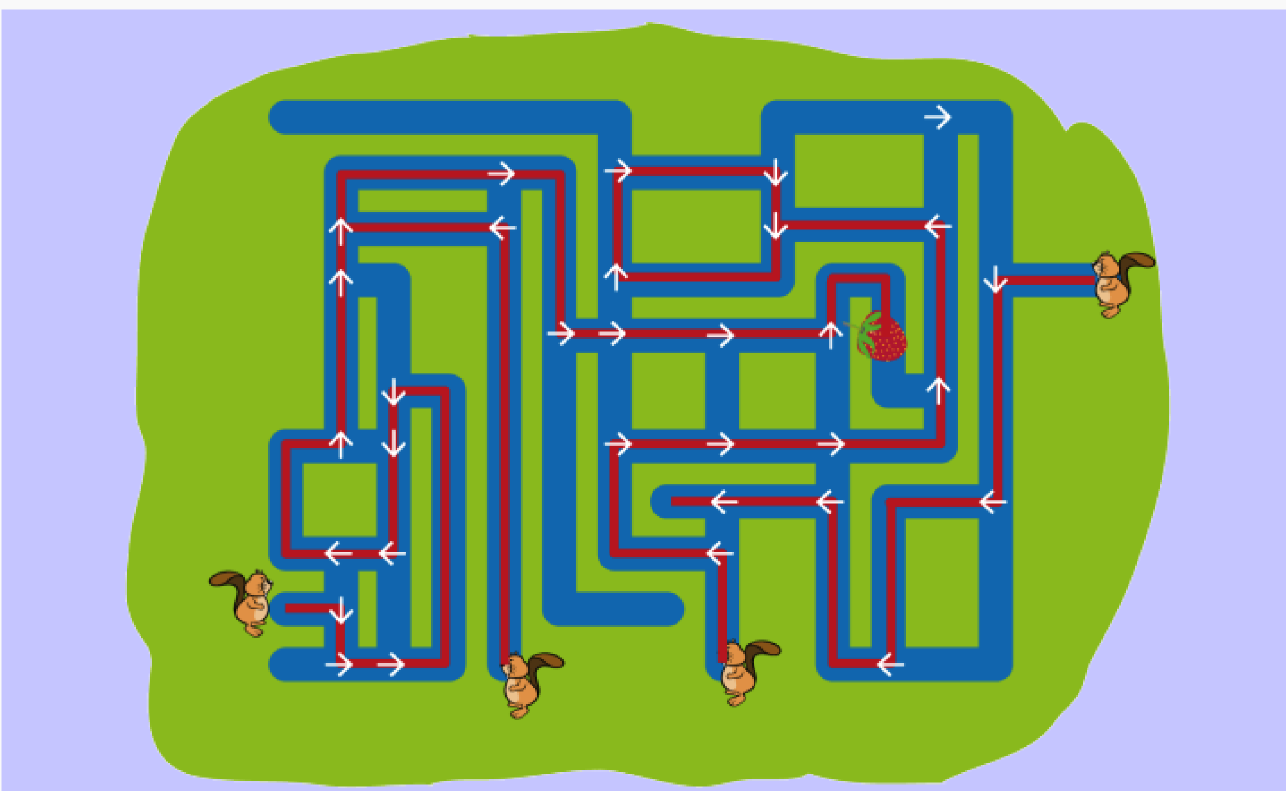
C. 3

D. 4

Solution

The answer is (B), two beavers find the strawberry.

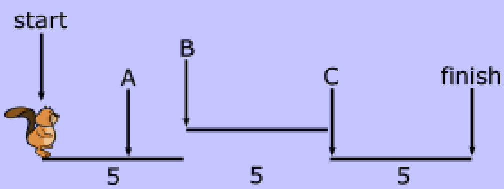
If we look at the paths of each of the beavers, we can see that it is the two beavers on the left that will reach the strawberry (indeed, their paths actually meet!). The third beaver at the bottom will start swimming in the circle, and the beaver on the very right swims into a dead end and has no way of getting out.



Jumpers

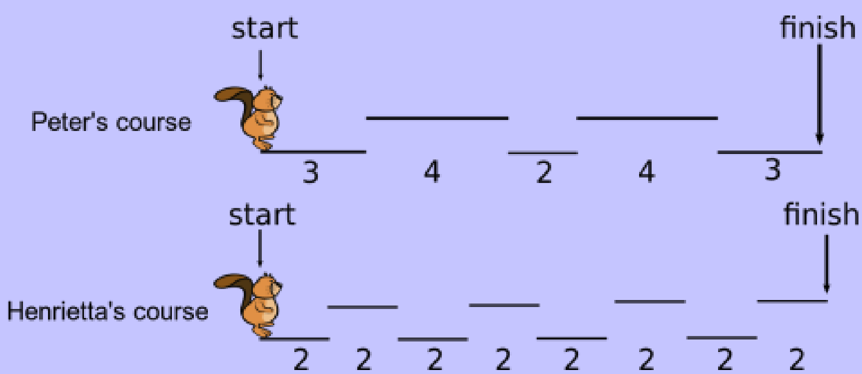
Peter and Henrietta are playing a video game. They move a beaver at a constant speed from the start of a course to the finish. The course consists of platforms on two levels. At the end of each platform before the finish, the beaver jumps instantaneously up or down to the next platform. The amount of time to move over each platform of the game is shown below each platform.

Here is an example course:



- 3 seconds after the start, the beaver is at A;
- 5 seconds after the start, the beaver is at B;
- 10 seconds after the start, the beaver is at C;
- 15 seconds after the start, the beaver is at the finish.

Peter and Henrietta start playing the following two different courses at exactly the same time.



For how long are both beavers moving along the top level at the same time?

A. 2 seconds

B. 4 seconds

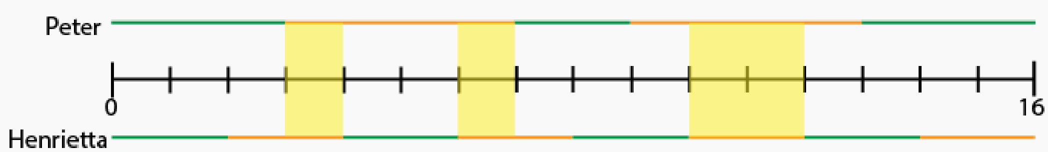
C. 6 seconds

D. 8 seconds

Solution

The answer is (B), both beavers are moving along the top level at the same time for a total of 4 seconds.

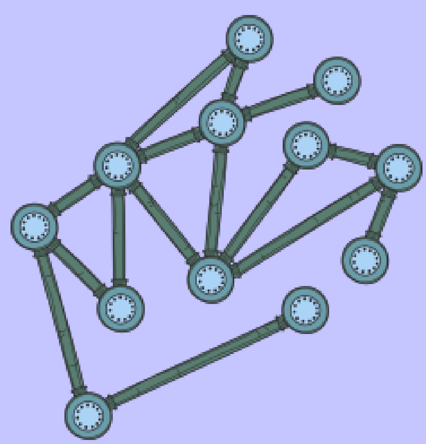
We can use a number line to show when Peter and Henrietta are moving along the bottom and top levels, and then look at when these times overlap:



If the green line represents moving along the bottom level and the orange line represents moving along the top level, we can see that there are three time slots where Peter and Henrietta's orange lines overlap. The first two times last 1 second each, and the third time lasts 2 seconds. So in total, there are 4 seconds when both beavers are moving along the top level at the same time.

Pipe Network

A network of 12 nodes connected by pipes is shown below. Exactly one node is clogged. However, even with this clog, water can flow between any pair of connected unclogged nodes in the network.



How many possibilities are there for the clogged node?

A. 5

B. 6

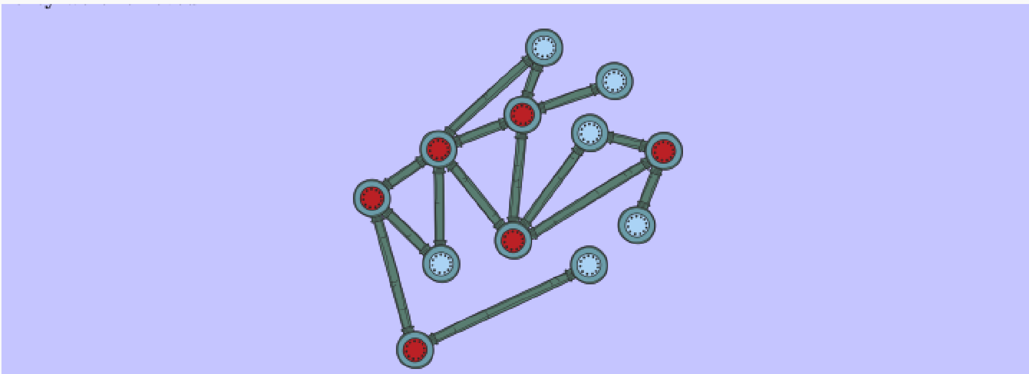
C. 7

D. 8

Solution

The answer is (B), there are six possibilities for the clogged node.

Looking at the diagram below, we can see that the six red nodes are the only ones which would disconnect the pipe network if they were removed. This leaves us with the other six nodes that would *not* disconnect the pipe network if they were removed.

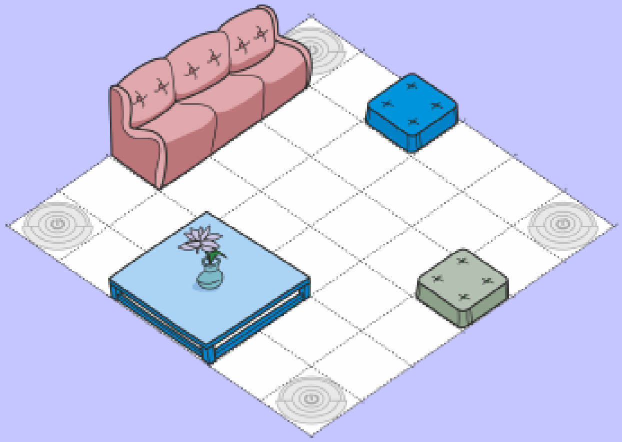


Robot Cleaner

A robot washes the square tiled floor shown below by using the following commands:

- F - move forward one tile (which takes 1 minute)
- W - wash a tile (which takes 1 minute)
- R - turn 90° right (which is performed instantly)
- L - turn 90° left (which is performed instantly)

The robot can start at any corner facing any direction and can end at any corner. It never goes on a tile occupied by one of the four pieces of furniture and washes all the other tiles, including the 4 corner tiles, exactly once. The robot may travel over a tile more than once.



What is the minimum possible number of minutes the robot needs to wash the entire floor?

A. 53

B. 54

C. 55

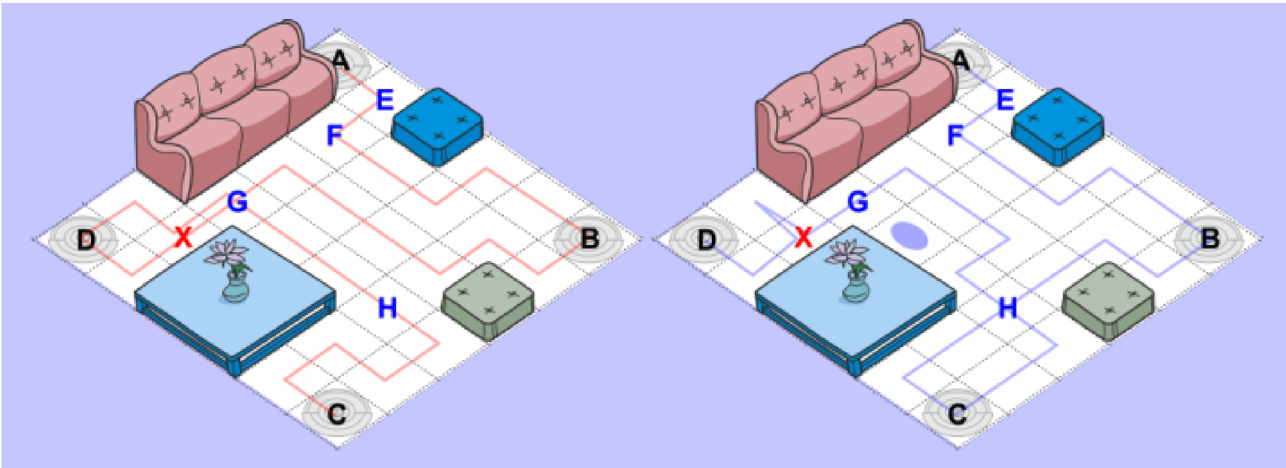
D. 56

Solution

The answer is (C), the robot needs a minimum of 55 minutes to wash the entire floor.

There are $36 - 9 = 27$ floor tiles, so washing takes 27 minutes. If every tile was visited once, moving would take 26 minutes (as we don't have to move to the first tile). So we have to minimize number of tiles which are passed twice.

Look at the pictures below. There is a tile X which must be passed twice every chosen way. Some of the tiles form a "narrow passage" to corner tiles A B C D: these tiles, labelled by the letters letters E, F, G, and H, must be passed over twice if the nearest corner to E, F, G, or H, is not a starting or ending tile. Tiles E and F are on the A narrow passage, tile G is on the D narrow passage, and H is on the C narrow passage. Corner B has no narrow passage tile. If robot starts or finishes at some corner, the narrow passage tile(s) on this corner can be passed through only once. Thus we can try to arrange the path so that robot went through as few narrow passage tiles as possible.



If the robot starts and finishes at the same corner tile it must pass all of the narrow passage tiles twice. We should thus make sure we start and end at different tiles to reduce our steps. As stated above, we want our robot to pass through as few narrow passage tiles as possible, and this includes trying to pass through them at most one time.Because corner A has two narrow passage tiles, E and F, a good strategy is to choose the corner A as a starting (or finishing) point. We then have to decide between finishing corners C and D (in both cases we spare another one narrow passage tiles of G or H).

Tracing the picture on the left shows that to move from A to C needs 28 moves forward (it contains points X and G). Tracing from A to D needs an odd number of moves so after using 28 moves forward at least one tile is not passed yet (in the right picture, one such tile which is not passed through yet is the one with the blue ellipse) so it is longer than from A to C.

Minimal time for cleaning is 27 (washing) + 28 (moving) minutes = 55 minutes.

Gauss Prep Solutions

1. The value of $4^2 - 2^3$ is

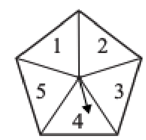
(A) 8 (B) 2 (C) 4 (D) 0 (E) 6

Solution

ANSWER: (A) 8

$4^2 - 2^3 = 16 - 8 = 8$

2. A pentagon is divided into 5 equal sections, as shown. An arrow is attached to the centre of the pentagon. The arrow is spun once. What is the probability that the arrow stops in the section numbered 4?

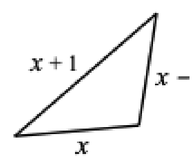


(A) $\frac{3}{5}$ (B) $\frac{1}{2}$ (C) $\frac{4}{5}$ (D) $\frac{1}{4}$ (E) $\frac{1}{5}$

Solution

ANSWER: (D) $\frac{1}{5}$

3. If the perimeter of the triangle shown is 21, what is the value of x?



(A) 3 (B) 7 (C) 8 (D) 13 (E) 16

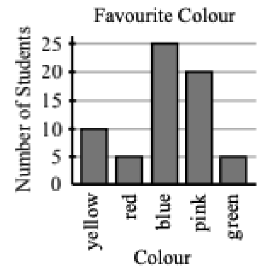
Solution

ANSWER: (B) 7

The perimeter of a shape is the sum of all its sides. This gives:

$$\begin{aligned}(x+1)+(x-1)+x &= 21\\ 3x &= 21\\ x &= 7\end{aligned}$$

4. Students were surveyed about their favourite colour and the results are displayed in the graph shown. What is the ratio of the number of students who chose pink to the number of students who chose blue?



(A) 4 : 5 (B) 3 : 5 (C) 1 : 5 (D) 2 : 5 (E) 5 : 3

Solution

ANSWER: (A) 4 : 5

If we look at the graph, we can see that 20 students chose pink and 25 students chose blue. Thus our ratio is 20 : 25, but we can reduce this further:

$20 \div 5 = 4$

$25 \div 5 = 5$

$\therefore\ 20 : 25 = 4 : 5$.

5. When a number is tripled and then decreased by 6, the result is 15. The number is

(A) 8 (B) 6 (C) 5 (D) 7 (E) 9

Solution

ANSWER: (D) 7

Let us call the number in question n and find an equivalent equation that represents the words described in the question:

- Tripled $\longrightarrow$ multiplication by 3 $\longrightarrow 3n$
- Decreased by 6 $\longrightarrow$ subtract 6 $\longrightarrow 3n - 6$
- Result is 15 $\longrightarrow$ result is equal to 15 $\longrightarrow 3n - 6 = 15$

We can now simply solve for n:

$$
\begin{aligned}
3n - 6 &= 15 \\
3n &= 15 + 6 \\
3n &= 21 \\
n &= 21 \div 3 \\
n &= 7
\end{aligned}
$$

6. The sum of the first 100 positive integers (that is, $1+2+3+\cdots+99+100$) equals 5050. The sum of the first 100 positive multiples of 10 (that is, $10+20+30+\cdots+990+1000$) equals

(A) 10 100 (B) 5950 (C) 50 500 (D) 6050 (E) 45 450

Solution

ANSWER: (C) 50 500

Notice that in $10+20+30+\cdots+990+1000$, each term is 10 times larger than its corresponding term in the sum $1+2+3+\cdots+99+100$. It then follows that the required sum is 10 times larger than the sum that has been given:

Since $1+2+3+\cdots+99+100 = 5050$, $10+20+30+\cdots+990+1000 = 5050 \times 10 = 50\,500$.

7. Brodie and Ryan are driving directly towards each other. Brodie is driving at a constant speed of 50 km/h. Ryan is driving at a constant speed of 40 km/h. If they are 120 km apart, how long will it take before they meet?

(A) 1h 12min (B) 1h 25min (C) 1h 15min (D) 1h 33min
(E) 1h 20min

Solution

ANSWER: (E) 1h 20min

When Brodie and Ryan are driving directly towards each other at their constant speeds of 50km/h and 40km/h, respectively, then the distance between them is decreasing at a rate of 90km/h ($50 + 40 = 90$).

If the initial distance between Brodie and Ryan is 120km, and the distance is decreasing at 90km/h, then they will meet after $\frac{120}{90}$ hours, or $1\frac{1}{3}$ hours.

$\frac{1}{3}$ of an hour is $\frac{1}{3} \times 60 = 20$ minutes, so it will take Brodie and Ryan 1 h 20 min to meet.

8. In a group of seven friends, the mean (average) age of three of the friends is 12 years and 3 months and the mean age of the remaining four friends is 13 years and 5 months. In months, the mean age of all seven friends is

(A) 156 (B) 154 (C) $155\frac{1}{2}$ (D) 157 (E) 155

Solution

ANSWER: (E) 155

If the average age of three of the friends is 147 months ($12 \times 12 + 3 = 147$) then the sum of their ages is $147 \times 3 = 441$ months.

If the average age of the other four friends is 161 months ($13 \times 12 + 5 = 161$) then the sum of their ages is $161 \times 4 = 644$ months.

The total sum of the ages of the seven friends is $441 + 644 = 1085$, and so the mean age of all seven friends is $1085 \div 7 = 155$ months.

9. Brady is stacking 600 plates in a single stack. Each plate is coloured black, gold or red. Any black plates are always stacked below any gold plates, which are always stacked below any red plates. The total number of black plates is always a multiple of two, the total number of gold plates is always a multiple of three, and the total number of red plates is always a multiple of six. For example, the plates could be stacked with:

- 180 black plates below 300 gold plates below 120 red plates, or
- 450 black plates below 150 red plates, or
- 600 gold plates.

In how many different ways could Brady stack the plates?

(A) 5139 (B) 5142 (C) 5145 (D) 5148 (E) 5151

Solution

ANSWER: (E) 5151

There are different ways that you could go about solving this question; we will be following the solution below:

In a given way of stacking the plates, let b be the number of groups of 2 black plates, g be the number of groups of 3 gold plates, and r be the number of groups of 6 red plates.

Then there are $2b$ black plates, $3g$ gold plates, and $6r$ red plates.

Since the total number of plates in a stack is 600, we have $2b + 3g + 6r = 600$.

We note that the numbers of black, gold and red plates completely determines the stack (we cannot rearrange the plates in any way), and so the number of ways of stacking the plates is the same as the number of ways of solving the equation $2b + 3g + 6r = 600$ where b, g, r are integers that are greater than or equal to 0.

Since r is at least 0 and $6r$ is at most 600, then the possible values for r are 0, 1, 2, 3, . . . , 98, 99, 100.

When $r = 0$, we obtain $2b + 3g = 600$.

Since g is at least 0 and $3g$ is at most 600, g is at most 200.

Because $2b$ and 600 are even, $3g$ is even, and so g is even.

Therefore, the possible values for g are 0, 2, 4, . . . , 196, 198, 200.

Since $200 = 100 \times 2$, there are 101 possible values for g.

When $g = 0$, we get $2b = 600$ and so $b = 300$.

When $g = 2$, we get $2b = 600 - 3 \times 2 = 594$ and so $b = 297$.

Each time we increase g by 2, the number of gold plates increases by 6, so the number of black plates must decrease by 6, and so b decreases by 3.

Thus, as we continue to increase g by 2s from 2 to 200, the values of b will decrease by 3s from 297 to 0.

In other words, every even value for g does give an integer value for b.

Therefore, when $r = 0$, there are 101 solutions to the equation.

When $r = 1$, we obtain $2b + 3g = 600 - 6 \times 1 = 594$.

Again, g is at least 0, is even, and is at most $594 \div 3 = 198$.

Therefore, the possible values of g are 0, 2, 4, . . . , 194, 196, 198.

Again, each value of g gives a corresponding integer value of b.

Therefore, when $r = 1$, there are 100 solutions to the equation.

Consider the case of an unknown value of r, which gives $2b + 3g = 600 - 6r$.
Again, g is at least 0 and is even.
Also, the maximum possible value of g is $\frac{600-6r}{3} = 200 - 2r$.
This means that there are $(100 - r) + 1 = 101 - r$ possible values for g. (Can you see why?)
Again, each value of g gives a corresponding integer value of b.
Therefore, for a general r between 0 and 100, inclusive, there are $101 - r$ solutions to the equation.
We make a table to summarize the possibilities:

r	g	b	# of solutions
0	0, 2, 4, . . ., 196, 198, 200	300, 297, 294, . . ., 6, 3, 0	101
1	0, 2, 4, . . ., 194, 196, 198	297, 294, 291, . . ., 6, 3, 0	100
2	0, 2, 4, . . ., 192, 194, 196	294, 291, 288, . . ., 6, 3, 0	99
⋮	⋮	⋮	⋮
98	0, 2, 4	6, 3, 0	3
99	0, 2	3, 0	2
98	0	0	1

Therefore, the total number of ways of stacking the plates is

$$101 + 100 + 99 + ... + 3 + 2 + 1$$

We note that the integers from 1 to 100 can be grouped into 50 pairs each of which has a sum of 101 ($1 + 100$, $2 + 99$, etc) $\therefore$ the number of ways that Brady could stack the plates is $101 + 50 \times 101 = 5151$.

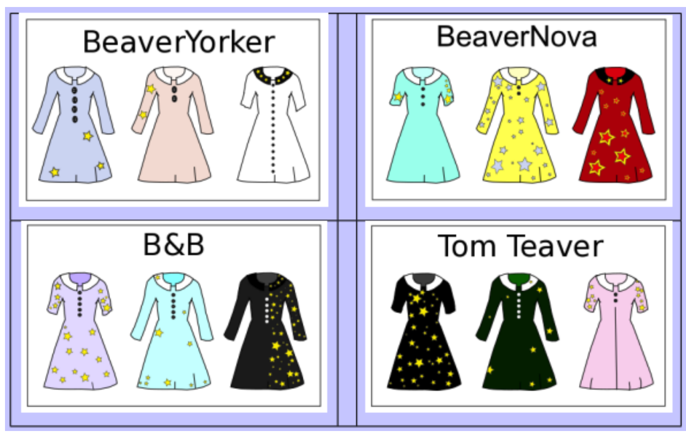
Problem Set Solutions

1. Dream Dress (BCC Grade 7/8 2015)

   Kate wants to buy her dream dress. It must

   - have short sleeves, and
   - have more than 3 buttons, and
   - have stars on its sleeves

   Four shops sell only the dresses shown:



   Which of these shops sells Kate's dream dress?

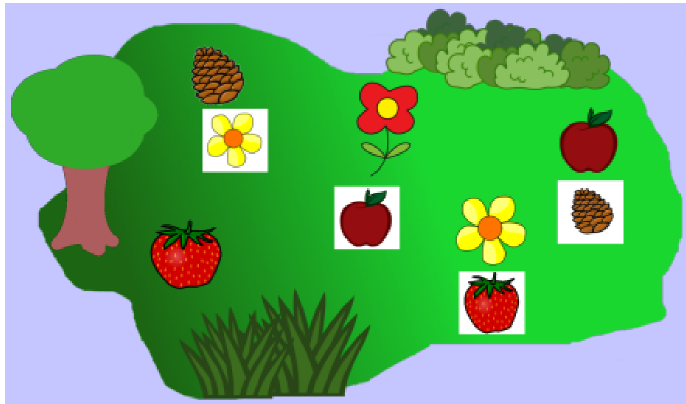
   (A) Beaver Yorker (B) Beaver Nova (C) B & B (D) Tom Teaver

   > *Solution*: (C) B & B
   > The light purple dress from the B & B shop is short sleeve, has 5 buttons, and has stars on its sleeves, so it satisfies Kate's dream dress.

2. Secret Recipe (BCC Grade 7/8 2016)

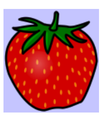
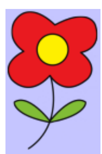
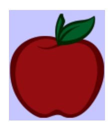
   Beavers are preparing for a Food Festival. They would like to bake a cake but their baker is on vacation. Keith decides to try to bake the cake. He remembers that it is important to add five essential ingredients in the correct order.

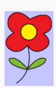
   When he gets to the garden shown below, he finds a white piece of paper beside all but one ingredient. The paper shows which ingredient must be added next.



So, for example, a yellow five-petal flower must be added immediately after a pine cone. And, since there is no paper beside the strawberry, it must be added last.

Which ingredient must be added first?

(A)  (B)  (C)  (D) 

Solution: (B) 

We can determine the answer by going backwards based on the pictures:

- It is given that the strawberry is last.
- The strawberry paper is beside the yellow flower, so the flower preceeds the strawberry.
- The yellow flower paper is beside the pinecone, so the pinecone preceeds the flower.
- The pinecone paper is beside the apple, so the apple preceeds the pinecone.
- The apple paper is beside the red flower, so the red flower preceeds the apple.

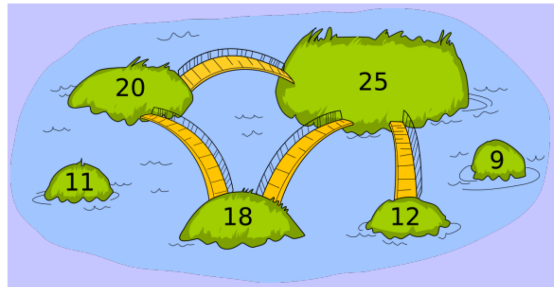
Since there is no paper with the red flower, there is no ingredient that preceeds it and so it must be first.

3. Connect the Islands (BCC Grade 7/8 2018)

People of Kastoria use only one rule to decide where bridges are to be built:

They choose one number called the bridge number. If the sum of the populations of two islands is greater than the bridge number, a bridge is built between the islands. Otherwise, a bridge is not built between the two islands.

The six islands of Kastoria and their populations are shown in the picture. The bridges built using the above rule are also shown.



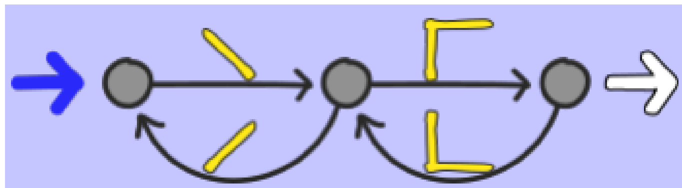
What bridge number was chosen?

(A) 34 (B) 35 (C) 36 (D) 37

Solution: (C) 36
The bridge number cannot be 34 or 35 because $11 + 25 = 36$ and there is not a bridge between the islands with populations of 11 and 25. The bridge number cannot be 37 because $12 + 25 = 37$ and there is a bridge between the islands with populations of 25 and 12. For each pair of islands connected by a bridge, the sum of their populations is greater than 36. That is, $18 + 20 = 38$, $18 + 25 = 43$, $20 + 25 = 45$ and $12 + 25 = 37$. Therefore the bridge number is 36.

4. Making Stitches (BCC 7/8 2019)

   A sewing machine can make four different types of stitches. The rules that the machine follows are shown.

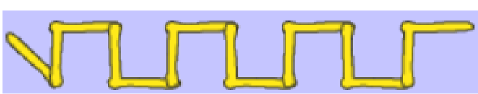


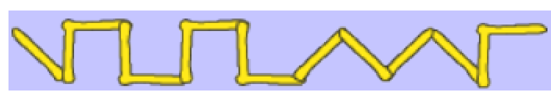
   The machine starts a new line of stitches by following the thick blue arrow on the left.

   Then the machine moves from circle to circle following in the direction of the arrows. Every time an arrow is followed, the machine makes the stitch shown on that arrow. If a circle has more than one arrow leading out of it, the user of the machine can choose to follow either one of the arrows.

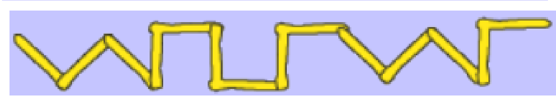
   The machine finishes by following the outlined arrow on the right.

   Which line of stitches **cannot** be made using the above rules?

(A) 

(B) 

(C) 

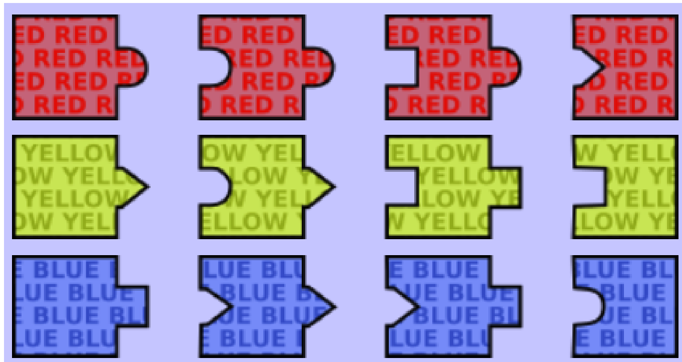
(D) 

Solution: (D) 

Following the machine rules, after every 'upside-down L'-stitch there is either no stitch, or an 'L'-stitch. The line of stitches for (D) has a diagonal stitch right after an 'upside-down L'-stitch, which cannot be made using our machine rules.

5. Puzzle Pieces (BCC 7/8 2020)

   A beaver has a puzzle with 12 different types of pieces, 4 of which are red, 4 of wich are yellow, and 4 of which are blue. There is an unlimited number of each type of piece, which is shown below.



Using these pieces, the beaver can create various colour sequences. The first piece in a sequence must have a flat left side and the last piece must have a flat right side. Pieces join in the usual way, but two pieces can't be joined on their flat sides and pieces can't be rotared. One possible sequence is shown below.

Which of the following colour sequences **cannot** be constructed?

(A) YELLOW → BLUE → BLUE → RED → BLUE

(B) BLUE → YELLOW → RED → YELLOW → RED

(C) RED → RED → YELLOW → BLUE → BLUE

(D) BLUE → RED → YELLOW → BLUE → RED

> *Solution*: (C) RED → RED → YELLOW → BLUE → BLUE
> This colour sequence has blue as the last puzzle piece, which needs to fit with a puzzle piece that has a rounded side. As the red puzzle pieces are the only ones with a roundded right side, we cannot have blue as the second-last colour in this sequence.

6. Missing Erasers (BCC 7/8 2021)

Four students were helping their teacher clean up. While cleaning, one of the students hid the blackboard erasers. When the teacher realized that the erasers were missing, she asked the students, "Which one of you hid the erasers?" Each student answered as follows:

Amélie: "I didn't hide the erasers."

Benin: "Dahila didn't hide the erasers."

Cai: "Amélie hid the erasers."

Dahila: "Either Benin or Cai hid the erasers."

Only one of these answers was true.

Which student hid the erasers?

(A) Amélie (B) Benin (C) Cai (D) Dahila

Solution: (D) Dahila hid the erasers.
We see that there are two contradicting statements from Amèlie and Cai: one states that Amèlie didn't hide the erasers, and the other states that she did. Evidently, they cannot both be true or both be false at the same time, which means that one of these statements **must** be true.
Let's assume that Cai was telling the truth. Then the other three answers are false, meaning that Benin lied. This means that "Dahila didn't hide the erasers" is false, implying that Dahlia *did* hide the erasers, but then Amelie would not have hidden the erasers and we come to a contradiction.
Let's assume then that Amèlie was telling the truth (she *did not* hide the erasers). Then the answers from Benin, Cai, and Dahila are false. If Benin was lying when he said that "Dahila didn't hide the erasers" then the truth would be that Dahila **did** hide the erasers. This does not contradict Cai's lie or Dahila's lie, so Dahila hid the erasers.

Gauss Problem Set

This problem set was created by the CEMC Problem Set Generator.

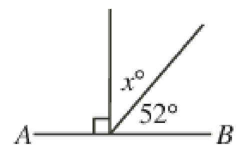
1. What time is it 45 minutes after 10:20?

   (A) 11:00 (B) 9:35 (C) 11:15 (D) 10:55 (E) 11:05

   (Source: 2015 Gauss (Grade 8), #2)

   Solution: (E) 11:05
   45 = 40 +5
   Since 11:00 is 40 minutes after 10:20, and 11:05 is 5 minutes after 11:00. 11:05 is 45 minutes after 10:20.

2. In the diagram, AB is a line segment. The value of x is



   (A) 128 (B) 38 (C) 48 (D) 142 (E) 308

   (Source: 2009 Gauss (Grade 8), #4)

   Solution: (B) 38
   A line segment has an angle of 180°. Since the square box represents 90°, it must be that:

$$\begin{aligned}90 + x + 52 &= 180\\ 142 + x &= 180\\ x &= 180 - 142\\ x &= 38\end{aligned}$$

3. Which of the following integers is closest to zero?

   (A) -1101 (B) 1011 (C) -1010 (D) -1001 (E) 1110

   (Source: 2014 Gauss (Grade 8), #5)

Solution: (D) -1001
We can simply compare the *absolute values* (ignore the negatives) of the integers, in which case -1001 is the smallest and so it is the closest to zero.

4. How many prime numbers are there between 20 and 30?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

   (Source: 2006 Gauss (Grade 8), #6)

Solution: (C) 2
The prime numbers between 20 and 30 are: 23, 29

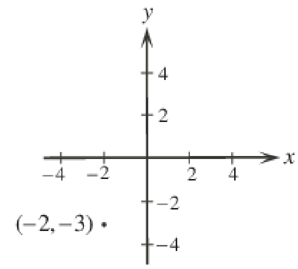
5. Yvon has 4 different notebooks and 5 different pens. He must bring exactly one notebook and exactly one pen to his class. How many different possible combinations of notebooks and pens could he bring?

   (A) 9 (B) 16 (C) 20 (D) 10 (E) 5

   (Source: 2020 Gauss (Grade 8), #7)

Solution: (C) 20
For each of Yvon's 4 notebooks, there are 5 pen possibilities. In other words, four groups of five combinations which gives us $4 \times 5 = 20$ total combinations.

6. The point (-2, -3) is reflected in the x-axis. What are the coordinates of its image after the reflection?



(A) (2, -3) (B) (3, -2) (C) (2,3) (D) (-3, -2)
(E) (-2, 3)

(Source: 2016 Gauss (Grade 8), #9)

Solution: (E) (-2, 3) For a point to be reflected in the x-axis means that it "flips" from being above the x-axis to being below, or vice-versa. This means that the x coordinate does not change, but the y coordinate changes its sign from positive to negativce, or vice-versa. In this case, our -3 would turn to a 3 and hence the coordinates of the image are (-2, 3).

7. The numbers 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 are written on separate cards and placed face down on a table. A card is chosen at random and flipped over. What is the probability that the number on this card is a prime number?

(A) $\frac{2}{11}$ (B) $\frac{4}{11}$ (C) $\frac{6}{11}$ (D) $\frac{3}{11}$ (E) $\frac{5}{11}$

(Source: 2008 Gauss (Grade 8), #10)

Solution: (E) $\frac{5}{11}$
There are five prime numbers among the cards (3, 5, 7, 11, 13) and eleven numbers in total, so the probability is $\frac{desired\ outcome}{total\ outcomes} = \frac{5}{11}$

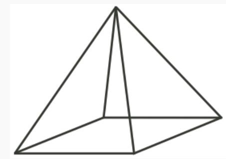
8. A pyramid has a square base. How many edges does the pyramid have?

(A) 8 (B) 6 (C) 12 (D) 5 (E) 3

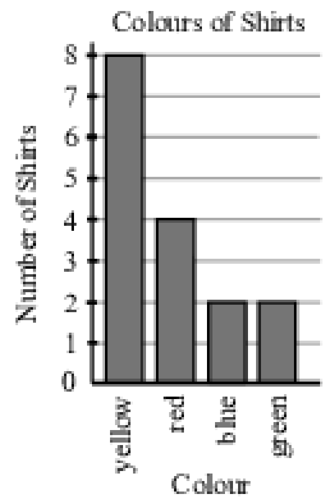
(Source: 2012 Gauss (Grade 8), #11)

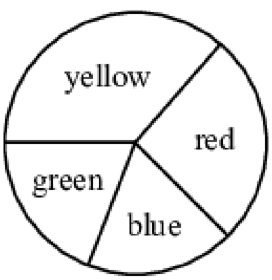
Solution: (A) 8

Since the pyramid has a square base, the base of the pyramid has 4 edges (one for each side of the square). An edge joins each of the 4 vertices of the square to the apex of the pyramid, as shown. In total, a pyramid with a square base has 8 edges.

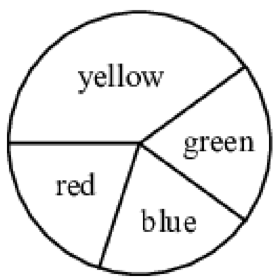


9. Which of the following circle graphs best represents the information in the bar graph shown?

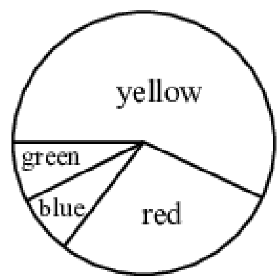




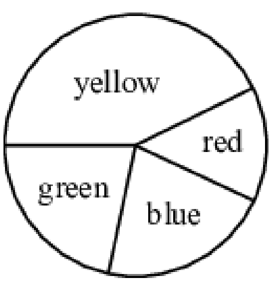
(A)



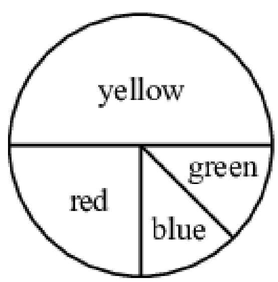
(B)



(C)



(D)



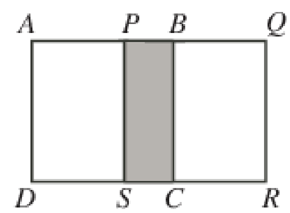
(E)

Solution: (E)
From the bar graph, we see that there are 8 yellow shirts, 4 red shirts, 2 blue shirts and 2 green shirts. In total, the number of shirts is $8 + 4 + 2 + 2 = 16$.

Thus the 8 yellow shirts represent $\frac{8}{16}$ or $\frac{1}{2}$ of the total number of shirts. The only circle graph showing that approximately half of the shirts are yellow is (E), so it most likely best represents the information from the bar graph.
We may confirm our choice by noting that this circle graph also shows that approximately $\frac{4}{16} = \frac{1}{4}$ of the shirts are red, approximately $\frac{2}{16} = \frac{1}{8}$ of the shirts are green and approximately $\frac{1}{8}$ of the shirts are blue.

(Source: 2021 Gauss (Grade 8), #13)

10. Two identical squares, $ABCD$ and $PQRS$ have side length 12. They overlap to form the 12 by 20 rectangle $AQRD$ shown. What is the area of the shaded rectangle $PBCS$?

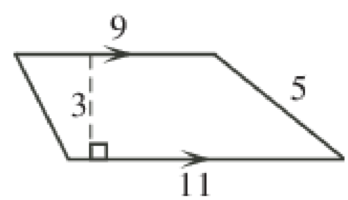


(A) 24 (B) 36 (C) 48 (D) 72 (E) 96

(Source: 2010 Gauss (Grade 8), #16)

Solution: (C) 48
Since $AQ = 20$ and $AB = 12$, then $BQ = AQ - AB = 20 - 12 = 8$.
Thus $PB = PQ - BQ = 12 - 8 = 4$.
Since $PS = 12$, the area of rectangle $PBCS$ is $12 \times 4 = 48$.

11. What is the area of the figure shown?



(A) 45 (B) 55 (C) 27 (D) 30 (E) 33

(Source: 2008 Gauss (Grade 8), #20)

Solution: (D) 30
The area of a trapezoid equals one-half multiplied by the sum of the bases, multiplied by the height. Therefore the area of this trapezoid is:

$$\frac{1}{2} \times (9 + 11) \times 3 = \frac{1}{2} \times 20 \times 3 = 10 \times 3 = 30$$

12. The product of four *different* positive integers is 360. What is the maximum possible sum of these four integers?

(A) 68 (B) 66 (C) 52 (D) 39 (E) 24

(Source: 2009 Gauss (Grade 8), #21)

Solution: (B) 66
For the sum to be a maximum, we must choose the three smallest divisors in an effort to makethe fourth divisor as large as possible.
The smallest 3 divisors of 360 are 1, 2, and 3, making $\frac{360}{1\times2\times3} = 60$ the fourth divisor.
We note that 1, 2, 3 are the smallest three different divisors of 360, meaning that it is not possible to use a divisor greater than 60.
Replacing the divisor 60 with a smaller divisor will decrease the sum of the four divisors.
We can verify this using the following reasoning:
The product of 3 different positive integers is always greater than or equal to the sum of the 3 integers. For instance, $1 \times 2 \times 4 = 8 > 1 + 2 + 4 = 7$.
The next largest divisor less than 60 is 45, thus the remaining three divisors would have a product of $360 \div 45 = 8$ and therefore having a sum that is less than or equal to 8. This would give a combined sum that is less than or equal to $45 + 8 = 53$, much less than the previous sum of $1 + 2 + 3 + 60 = 66$. In the same way, we obtain sums smaller than 66 if we consider the other divisors of 360 as the largest of the four integers. Therefore the maximum possible sum is 66.

13. Greg, Charlize, and Azarah run at different but constant speeds. Each pair ran a race on a track that measured 100 m from start to finish. In the first race, when Azarah crossed the finish line, Charlize was 20 m behind. In the second race, when Charlize crossed the finish line, Greg was 10 m behind. In the third race, when Azarah crossed the finish line, how many metres was Greg behind?

(A) 20 (B) 25 (C) 28 (D) 32 (E) 40

(Source: 2013 Gauss (Grade 8), #23)

Solution: (C) 28
In the first place, when Azarah crossed the finish line, Charlize had run 80m (as she was 20m behind). Since Azarah and Charlize travelled their respective distances in the same amount of time, the ratio of their speeds is equal to the ratio of their distances travelled. That is, Charlize's speed is 80% of Azarah's speed.
Similarly, Greg's speed is 90% of Charlize's speed.
Therefore Greg's speed with Azarah is 90% of Charlize's speed with respect to Azarah, which is 80%. Since 90% of 80% is equal to $0.9 \times 0.8 = 0.72 = 72\%$, Greg's speed is 74% of Azarah's speed.
This means that in the third race, when Azarah ran 100m, Greg ran 72% of 100m or 72m in the same amount of time, and so Greg was $100 - 72 = 28$m behind Azarah.

14. What is the tens digit of 3^{2016}?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

(Source: 2016 Gauss (Grade 8), #24)

Solution: (B) 2
When two integers are multiplied together, the final two digits (the tens digit and the units digits) of the product are determined by the final two digits of each of the two numbers that are multiplied. This is true since the place value of any digit contributes to its equal place value (and possibly also to a greater place value) in the product. That is, the hundreds digit of each number being multiplied contributes to the hundreds digit (and possibly to digits of higher place value) in the product. Thus, to determine the tens digit of any product, we need only consider the tens digits and the units digits of each of

the two numbers that are being multiplied.
For example, to determine the final two digits of the product 1215×603 we consider the product of $15 \times 03 = 45$. We may verify that the tens digit of the product $1215 \times 603 = 732645$ is indeed 4 and the units digit is indeed 5.
Since $3^5 = 243$, the final two digits of $3^{10} = 3^5 \times 3^5 = 243 \times 243$ are given by the product $43 \times 43 = 1849$ and thus are 49.
Since the final two digits of 3^{10} are 49 and $3^{20} = 3^{10} \times 3^{10}$, the final two digits of 3^{20} are given by $49 \times 49 = 2401$ and hence are 01.
Similarly, $3^{40} = 3^{20} \times 3^{20}$ ends in 01 (since $01 \times 01 = 01$).
Further, 3^{20} multiplied by itself 100 times, or $(3^{20})^{100} = 3^{2000}$, also ends with 01.
As seen above, 3^{10} ends with 49 and 3^5 ends with 43, so then $3^{15} = 3^{10} \times 3^5$ ends with $49 \times 43 = 2107$ giving it final two digits of 01.
This tells us that $3^{16} = 3^{15} \times 3^1$ ends with $07 \times 03 = 21$.
Putting all of this together, we have $3^{2016} = 3^{2000} \times 3^{16}$ and thus ends in $01 \times 21 = 21$, and so the tens digit of 3^{2016} is 2.

15. Daryl first writes the perfect squares as a sequence

$$1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \ldots$$

After the number 1, he then alternates by making two terms negative followed by leaving two terms positive. Daryl's new sequence is:

$$1, -4, -9, 16, 25, -36, -49, 64, 81, -100, \ldots$$

What is the sum of the first 2011 terms in this new sequence?

(A) -4 042 109 (B) -4 047 638 (C) -4 038 094 (D) -4 044 121 (E) -4 046 132

(Source: 2011 Gauss (Grade 8), #25)

Solution: (E) -4 046 132
The given sequence allows for many different patterns to be discovered depending on how terms in the sequence are grouped and then added. One possibility is to add groups of

four consecutive terms in the sequence. That is, consider finding the sum of the sequence, S, in the manner shown below.

$S = (1+(-4)+(-9)+16)+(25+(-36)+(-49)+64)+(81+(-100)+(-121)+144)+\dots$

The pattern that appears when grouping terms in this way is that each consecutive group of 4 terms, beginning at the first term, adds to 4.

That is, $1+(-4)+(-9)+16 = 4$, $25+(-36)+(-49)+64 = 4$, $81+(-100)+(-121)+144 = 4$, and so on.

For now, we will assume that this pattern of four consecutive terms adding to 4 continues and wait to verify this at the end of the solution. Since each consecutive group of four terms adds to 4, the first eight terms add to 8, the first twelve terms add to 12 and the first n terms add to n provided that n is a multiple of 4.

Thus, the sum of the first 2012 terms is 2012, since 2012 is a multiple of 4.

Since we are required to find the sum of the first 2011 terms, we must subtract the value of the 2012^{th} term from our total of 2012.

We know that the n^{th} term in the sequence is either n^2 or it is $-n^2$, therefore we must determine if the 2012^{th} term is positive or negative.

By the alternating pattern of the signs, the first and fourth terms in each of the consecutive groupings will be positive, while the second and third terms are negative.

Since the 2012^{th} term is fourth in its group of four, its sign is positive, so the 2012^{th} term is 2012^2.

Therefore the sum of the first 2011 terms is the sum of the sum of the first 2012 terms minus the 2012^{th} term, or:

$S = 2012 - 2012^2 = 2012 - 4048144 = -4046132$.

See the 2011 Gauss Solution to see how the pattern is verified.